ADOMIAN DECOMPOSITION APPROACH TO A SACRM TOILET INFECTION EPIDEMIC MODEL

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Abstract

In this paper, we provided a very accurate, non-perturbed, non-linearized and semi-analytic solution to a system of nonlinear first-order ordinary differential equations (ODEs) modeling the dynamics of toilet infections in Nigeria. Adomian decomposition method (ADM) is employed to compute an approximate solution of the system of ODEs governing the problem. Graphical results are presented and discussed quantitatively to illustrate the solution.

Keywords: Toilet infections, Nonlinear ordinary differential equation, Adomian decomposition method, Taylor's series, Epidemic model.

1. INTRODUCTION

> *"Viruses can survive on human hands for several hours and may be spread by direct contact. Given that we can touch up to 300 surfaces in half an hour, a person may easily pick up infections on their fingers by touching an infected object"* – Professor John Oxford of the Royal London Hospital

Using toilets in an unhygienic manner exposes one to the risk of contracting infections from the toilet. Unhealthy toilets are comfortable homes for disease-causing bacteria (microbes) and viruses. Toilet infection is different from sexuality transmitted infection (STI) in that STI can be transmitted through unprotected sexual contacts while toilet infection can be contracted through exposure to dirty toilets. Common toilet infections in Nigeria include Escherichia (Ecoli), Staphylococcus, Norovirus, Streptococcus, Gardnerella, Shigella Bacteria, and Influenza (Flu) (WTO, 2016). According to UNICEF (2012) statistics, in Nigeria, it is estimated that diarrhoea kills about 194,000 children under five every year and in addition, respiratory infections kill another 240,000. These are largely preventable with improvements in water, sanitation and hygiene.

Answering the call of nature can occur at any time and any place. Hence, we cannot always have the freedom to choose where to use toilets. For example, if you find yourself in a motor park and you are seriously pressed to use the toilet, you may not have the choice than to use the toilet they have in whatsoever condition you meet it. For this reason, we need to maintain good toilet hygiene. In order to find an efficient way to control an infection, it is of great importance to establish its transmission dynamics. One main goal of mathematical epidemiology is to understand how to control and eradicate diseases (Dahari *et al.*, 2009). There have been many literatures on the terrible state of Nigerian toilets and other toilets in the world and Yusuf, Bawa and Mayaki (2017) developed the first deterministic model for the control of toilets infections in Nigeria.

2. MATHEMATICAL FORMULATION

In this paper, the following toilet infections model proposed by Yusuf, Bawa, and Mayaki (2017) is tested to show the efficacy of the Adomian Decomposition Method to solve such model:

$$\frac{dS}{dt}=\Lambda-\frac{pc(1-\varepsilon)MS}{N}+\omega R-\mu S \tag{1}$$

$$\frac{dA}{dt}=\frac{pc(1-\varepsilon)MS}{N}-\left[\sigma_1(1-\rho)+\theta(1-\alpha)+\tau_1\alpha+\mu\right]A \tag{2}$$

$$\frac{dC}{dt}=\theta(1-\alpha)A-\left[\sigma_2(1-\rho)+\tau_2+\mu\right]C \tag{3}$$

$$\frac{dR}{dt}=\tau_1\alpha A+\tau_2 C-(\omega+\mu)R \tag{4}$$

$$\frac{dM}{dt}=\sigma_1(1-\rho)A+\sigma_2(1-\rho)C-\eta M \tag{5}$$

Where

$$N=S+A+C+R \tag{6}$$

So that

$$\frac{dN}{dt}=\Lambda-\mu N \tag{7}$$

in the biological feasible region

$$\Omega=\left\{S,A,C,R\in R_+^4 : M\in R_+^1\right\} \tag{8}$$

Table 1: Definition of Variables and Parameters of the Model

Variable/Parameter	Interpretation
$S(t)$	Susceptible individuals at time, t
$A(t)$	Acutely infected individuals at time, t
$C(t)$	Chronically infected individuals at time, t
$R(t)$	Recovered individuals at time, t
$M(t)$	Population of microbes in toilets at time, t
Λ	Recruitment number of human
p	The likelihood of being infected when there is a contact with an infected toilet
c	The average total contact with toilets
τ_1	Treatment rate of acutely infected individuals

τ_2	Treatment rate of chronically infected individuals
σ_1	Shedding rate of acutely infected individuals to the population of microbes in the environment
σ_2	Shedding rate of chronically infected individuals to the population of microbes in the environment
θ	Period from acute infection to chronic infection
ω	Rate of losing drug-induced immunity
μ	Per capita natural death rate of human
η	Mortality rate of the microbes
ε	Impact of public enlightenment campaign on the dangers of using dirty toilets, with $0<\varepsilon<1$
ρ	Rate of compliance to personal hygiene, with $0<\rho<1$
α	Proportion of acutely infected individuals receiving treatment

3. ADOMIAN DECOMPOSITION METHOD (ADM)

The decomposition method was first introduced and developed by Adomian (1994). It has been receiving much attention from researchers in recent years in the field of applied mathematics, in general, and in the area of initial and boundary value problems in particular. The method efficiently handles a wide class of linear/nonlinear ordinary and partial differential equations, linear and nonlinear integral equations, and integro-differential equations. The ADM provides several significant advantages; it demonstrates fast convergence of the solution. It also handles the problem in a direct way without using linearization, perturbation, or any other restrictive assumptions that may change the physical behavior of the model under study. Furthermore, it provides an efficient numerical solution in the form of an infinite series that is obtained iteratively and usually converges to the exact solution using Adomian polynomials. This method was used by Biazar (2006) and Makinde (2007) to solve epidemic models.

Consider the following equation:

$$Pu+Qu+Lu=g \tag{9}$$

where P is a linear operator, Q represents a nonlinear operator and L is the remaining linear part. Assuming that P is invertible, we define its inverse as P^{-1} so that

$$u=P^{-1}g-P^{-1}Qu-P^{-1}Lu \tag{10}$$

The Adomian Decomposition Method assumes that the unknown function, u can be expressed by an infinite series of the form:

$$u=\sum_{n=0}^{\infty}u_n=u_0+u_1+u_2+u_3+\ldots \tag{11}$$

where the components, u_n will be determined recursively. Moreover, the method defines the nonlinear term by the Adomian polynomials. More precisely, the ADM assumes that the nonlinear operator $Q(n)$ can be decomposed by an infinite series of polynomials given by

$$Q(u)=\sum_{n=0}^{\infty}F_n \tag{12}$$

where F_n are the Adomian's polynomials defined as

$$F_n = F_n(u_0, u_1, u_2, u_3, ..., u_n) \quad (13)$$

Substituting (11) into (10) and using the fact that L is a linear operator, we have

$$\sum_{n=0}^{\infty} u_n = P^{-1}g - P^{-1}\left(\sum_{n=0}^{\infty} F_n(u_0, u_1, u_2, u_3, ..., u_n)\right) - P^{-1}\left(\sum_{n=0}^{\infty} L(u_n)\right) \quad (14)$$

Therefore, the formal recurrence algorithm could be defined by

$$\left.\begin{array}{l} u_0 = P^{-1}g \\ u_1 = -P^{-1}\{F_0(u_0)\} - P^{-1}\{L(u_0)\} \\ u_2 = -P^{-1}\{F_1(u_0, u_1)\} - P^{-1}\{L(u_1)\} \\ u_3 = -P^{-1}\{F_2(u_0, u_1, u_2)\} - P^{-1}\{L(u_2)\} \\ u_{n+1} = -P^{-1}\{F_1(u_0, u_1, u_2, ..., u_n)\} - P^{-1}\{L(u_n)\} \\ ... \end{array}\right\} \quad (15)$$

Consider the non-linear function $f(u)$. Then, the infinite series generated by applying the Taylor's series expansion of f about the initial function u_0 is given by

$$f(u) = f(u_0) + f'(u_0)(u - u_0) + \frac{1}{2!} f''(u_0)(u - u_0)^2 + ... \quad (16)$$

Substituting (11) into (16), we have

$$f(u) = f(u_0) + f'(u_0)(u_1 + u_2 + u_3 + ...) + \frac{1}{2!} f''(u_0)(u_1 + u_2 + u_3 + ...)^2 + ... \quad (17)$$

Rearranging the terms in the expansion (17) according to the order, we have

$$\left.\begin{array}{l} f(u) = f(u_0) + f'(u_0)u_1 + f'(u_0)u_2 + \frac{1}{2!} f''(u_0)u_1^2 + f'(u_0)u_3 \\ + \frac{2}{2!} f''(u_0)u_1u_2 + \frac{1}{3!} f'''(u_0)u_1^3 + f'(u_0)u_4 + \frac{1}{2!} f''(u_0)u_2^2 \\ + \frac{2}{2!} f''(u_0)u_1u_3 + \frac{3}{3!} f'''(u_0)u_1^2u_2 + \frac{1}{4!} f''''(u_0)u_1^4 + ... \end{array}\right\} \quad (18)$$

The Adomian polynomials are constructed in a certain way so that the polynomial F_1 consists of all terms in the expansion (18) of order 1, F_2 consists of all terms of order 2, and so on. In general, F_n consists of all terms of order n. Therefore, the first nine terms of Adomian polynomials are listed as follows:

$$F_0 = f(u_0) \quad (19)$$

$$F_1 = f'(u_0)u_1 \quad (20)$$

$$F_2 = f'(u_0)u_2 + \frac{1}{2!} f''(u_0)u_1^2 \quad (21)$$

$$F_3 = f'(u_0)u_3 + \frac{2}{2!} f''(u_0)u_1u_2 + \frac{1}{3!} f'''(u_0)u_1^3 \quad (22)$$

$$F_4 = \left\{\begin{array}{l} f'(u_0)u_4 + \frac{1}{2!} f''(u_0)(2u_1u_3 + u_2^2) \\ + \frac{3}{3!} f'''(u_0)u_1^2u_2 + \frac{1}{4!} f^{(4)}(u_0)u_1^4 \end{array}\right\} \quad (23)$$

$$F_5 = \left\{ \begin{aligned} & f'(u_0)u_5 + \frac{1}{2!} f''(u_0)(2u_1u_4 + 2u_2u_3) \\ & + \frac{1}{3!} f'''(u_0)(3u_1^2u_3 + 3u_1u_2^2) \\ & + \frac{4}{4!} f^{(4)}(u_0)u_1^3u_2 + \frac{1}{5!} f^{(5)}(u_0)u_1^5 \end{aligned} \right\} \tag{24}$$

$$F_6 = \left\{ \begin{aligned} & f'(u_0)u_6 + \frac{1}{2!} f''(u_0)(2u_1u_5 + 2u_2u_4 + u_3^2) \\ & + \frac{1}{3!} f'''(u_0)(3u_1^2u_4 + u_2^3 + 6u_1u_2u_3) \\ & + \frac{1}{4!} f^{(4)}(u_0)(4u_1^3u_3 + 6u_1^2u_2^2) \\ & + \frac{5}{5!} f^{(5)}(u_0)u_1^4u_2 + \frac{1}{6!} f^{(6)}(u_0)u_1^6 \end{aligned} \right\} \tag{25}$$

$$F_7 = \left\{ \begin{aligned} & f'(u_0)u_7 + \frac{1}{2!} f''(u_0)(2u_1u_6 + 2u_2u_5 + 2u_3u_4) \\ & + \frac{1}{3!} f'''(u_0)(3u_1^2u_5 + 3u_1u_2^3 + 3u_3u_2^2 + 6u_1u_2u_4) \\ & + \frac{1}{4!} f^{(4)}(u_0)(4u_1^3u_4 + 12u_1^2u_2u_3 + 4u_1u_2^3) \\ & + \frac{1}{5!} f^{(5)}(u_0)(5u_1^4u_3 + 10u_1^3u_2^2) \\ & + \frac{1}{6!} f^{(6)}(u_0)u_1^5u_2 + \frac{1}{7!} f^{(7)}(u_0)u_1^7 \end{aligned} \right\} \tag{26}$$

$$F_8 = \left\{ \begin{aligned} & f'(u_0)u_8 + \frac{1}{2!} f''(u_0)(2u_1u_7 + 2u_2u_6 + 2u_3u_5 + u_4^2) \\ & + \frac{1}{3!} f'''(u_0)(3u_1^2u_6 + 3u_2^2u_4 + 3u_2u_3^2 + 6u_1u_2u_5 + 6u_1u_3u_4) \\ & + \frac{1}{4!} f^{(4)}(u_0)(4u_1^3u_5 + 12u_1^2u_2u_4 + 12u_1u_2^2u_3 + 6u_1^2u_3^2 + u_2^4) \\ & + \frac{1}{5!} f^{(5)}(u_0)(5u_1^4u_4 + 20u_1^3u_2u_3 + 10u_1^2u_2^3) \\ & + \frac{1}{6!} f^{(6)}(u_0)(u_1^5u_3 + 15u_1^4u_2^2) + \frac{7}{7!} f^{(7)}(u_0)u_1^6u_2 + \frac{1}{8!} f^{(8)}(u_0)u_1^8 \end{aligned} \right\} \tag{27}$$

4. SOLUTION OF THE SACRM TOILET INFECTION EPIDEMIC MODEL

The equivalent canonical form of system (1) – (5) is as follows:

$$S(t)=S(0)+\Lambda t-\frac{pc(1-\varepsilon)}{N}\int_0^t MSdt+\omega\int_0^t Rdt-\mu\int_0^t Sdt \tag{28}$$

$$A(t)=S(0)+\frac{pc(1-\varepsilon)}{N}\int_0^t MSdt-\left[\sigma_1(1-\rho)+\theta(1-\alpha)+\tau_1\alpha+\mu\right]\int_0^t Adt \tag{29}$$

$$C(t)=C(0)+\theta(1-\alpha)\int_0^t Adt-\left[\sigma_2(1-\rho)+\tau_2+\mu\right]\int_0^t Cdt \tag{30}$$

$$R(t)=R(0)+\tau_1\alpha\int_0^t Adt+\tau_2\int_0^t Cdt-(\omega+\mu)\int_0^t Rdt \tag{31}$$

$$M(t)=M(0)+\sigma_1(1-\rho)\int_0^t Adt+\sigma_2(1-\rho)\int_0^t Cdt-\eta\int_0^t Mdt \tag{32}$$

As usual in ADM, the solutions of (28) – (32) are considered to be as the sum of the following series:

$$\left.\begin{aligned} S&=\sum_{n=0}^{\infty}S_n\\ A&=\sum_{n=0}^{\infty}A_n\\ C&=\sum_{n=0}^{\infty}C_n\\ R&=\sum_{n=0}^{\infty}R_n\\ M&=\sum_{n=0}^{\infty}M_n \end{aligned}\right\} \tag{33}$$

We then approximate the non-linear terms in the system as follows

$$MS=\sum_{n=0}^{\infty}F_n\left(M_0,M_1,M_2,\ldots M_n,S_0,S_1,S_2,\ldots S_n\right) \tag{34}$$

Where F_n is the non-linear functions called the Adomian's polynomials and is given by

$$F_n=\frac{1}{n!}\left[\frac{d^n\left(\sum_{k=0}^{\infty}M_k\lambda^k\right)\left(\sum_{k=0}^{\infty}S_k\lambda^k\right)}{d\lambda^n}\right]_{\lambda=0} \tag{35}$$

Substituting (33) – (35) into (28) – (33) gives

$$\sum_{n=0}^{\infty} S_n = S(0) + \Lambda t - \frac{pc(1-\varepsilon)}{N}\int_0^t \sum_{n=0}^{\infty} F_n dt + \omega \int_0^t \sum_{n=0}^{\infty} R_n dt - \mu \int_0^t \sum_{n=0}^{\infty} S_n dt \tag{36}$$

$$\sum_{n=0}^{\infty} A_n = S(0) + \frac{pc(1-\varepsilon)}{N}\int_0^t \sum_{n=0}^{\infty} F_n dt - \left[\sigma_1(1-\rho) + \theta(1-\alpha) + \tau_1\alpha + \mu\right]\int_0^t \sum_{n=0}^{\infty} A_n dt \tag{37}$$

$$\sum_{n=0}^{\infty} C_n = C(0) + \theta(1-\alpha)\int_0^t \sum_{n=0}^{\infty} A_n dt - \left[\sigma_2(1-\rho) + \tau_2 + \mu\right]\int_0^t \sum_{n=0}^{\infty} C_n dt \tag{38}$$

$$\sum_{n=0}^{\infty} R_n = R(0) + \tau_1\alpha\int_0^t \sum_{n=0}^{\infty} A_n dt + \tau_2\int_0^t \sum_{n=0}^{\infty} C_n dt - (\omega+\mu)\int_0^t \sum_{n=0}^{\infty} R_n dt \tag{39}$$

$$\sum_{n=0}^{\infty} M_n = M(0) + \sigma_1(1-\rho)\int_0^t \sum_{n=0}^{\infty} A_n dt + \sigma_2(1-\rho)\int_0^t \sum_{n=0}^{\infty} C_n dt - \eta\int_0^t \sum_{n=0}^{\infty} M_n dt \tag{40}$$

From (36) – (40), we define the following scheme:

$$\left.\begin{aligned} S_0 &= S(0) + \Lambda t \\ S_{n+1} &= -\frac{pc(1-\varepsilon)}{N}\int_0^t F_n dt + \omega\int_0^t R_n dt - \mu\int_0^t S_n dt, \ (for\, n \geq 0) \end{aligned}\right\} \tag{41}$$

$$\left.\begin{aligned} A_0 &= S(0) \\ A_{n+1} &= \frac{pc(1-\varepsilon)}{N}\int_0^t F_n dt - \left[\sigma_1(1-\rho) + \theta(1-\alpha) + \tau_1\alpha + \mu\right]\int_0^t A_n dt, \ (for\, n \geq 0) \end{aligned}\right\} \tag{42}$$

$$\left.\begin{aligned} C_0 &= C(0) \\ C_{n+1} &= \theta(1-\alpha)\int_0^t A_n dt - \left[\sigma_2(1-\rho) + \tau_2 + \mu\right]\int_0^t C_n dt, \ (for\, n \geq 0) \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned} R_0 &= R(0) \\ R_{n+1} &= \tau_1\alpha\int_0^t A_n dt + \tau_2\int_0^t C_n dt - (\omega+\mu)\int_0^t R_n dt, \ (for\, n \geq 0) \end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned} M_0 &= M(0) \\ M_{n+1} &= \sigma_1(1-\rho)\int_0^t A_n dt + \sigma_2(1-\rho)\int_0^t C_n dt - \eta\int_0^t M_n dt, \ (for\, n \geq 0) \end{aligned}\right\} \tag{45}$$

Using (35), we compute some of the Adomian polynomials as follows:

$$F_0 = M_0 S_0 \tag{46}$$

$$F_1 = M_0 S_1 + M_1 S_0 \tag{47}$$

$$F_2 = M_0S_2 + M_1S_1 + M_2S_0 \tag{48}$$

$$F_3 = M_0S_3 + M_1S_2 + M_2S_1 + M_3S_0 \tag{49}$$

$$F_4 = M_0S_4 + M_1S_3 + M_2S_2 + M_3S_1 + M_4S_0 \tag{50}$$

$$F_5 = M_0S_5 + M_1S_4 + M_2S_3 + M_3S_2 + M_4S_1 + M_5S_0 \tag{51}$$

$$F_6 = M_0S_6 + M_1S_5 + M_2S_4 + M_3S_3 + M_4S_2 + M_5S_1 + M_6S_0 \tag{52}$$

$$\vdots$$

Substituting (46) – (52) into (36) – (40) and using Maple computational software, we obtain a few terms approximation to the solution as

$$\left.\begin{aligned} S_N &= \sum_{n=0}^{N} S_n \\ A_N &= \sum_{n=0}^{N} A_n \\ C_N &= \sum_{n=0}^{N} C_n \\ R_N &= \sum_{n=0}^{N} R_n \\ M_N &= \sum_{n=0}^{N} M_n \end{aligned}\right\} \tag{53}$$

Where

$$\left.\begin{aligned} S(t) &= \lim_{N\to\infty}(S_N) \\ A(t) &= \lim_{N\to\infty}(A_N) \\ C(t) &= \lim_{N\to\infty}(C_N) \\ R(t) &= \lim_{N\to\infty}(R_N) \\ M(t) &= \lim_{N\to\infty}(M_N) \end{aligned}\right\} \tag{54}$$

6. NUMERICAL RESULTS AND DISCUSSION

For the numerical results, the following values for variables and parameters are considered:

Table 2: Variables and population-dependent parameter values

S/N	Variable	Total Number
1	S	142,028,910
2	A	33,024,476
3	C	10,000,000
4	R	1,000,000
5	M	200,000,000
6	N	186,053,386
7	Λ	2,790,800

Source: Yusuf, Bawa, and Mayaki (2017)

Table 3: Values of population-independent parameter

S/N	Parameter	Value
1	p	0.75
2	c	6
3	τ_1	0.75
4	τ_2	0.45
5	σ_1	0.02
6	σ_2	0.04
7	θ	0.2
8	ω	0.164
9	μ	0.015
10	η	0.0001
11	ε	(0,1)
12	ρ	(0,1)
13	α	(0,1)

Source: Yusuf, Bawa, and Mayaki (2017)

Substituting the initial values in Table 2 and Table 3 in (41) – (45) and considering (46) – (52), using Maple computational software for high degree of accuracy, we obtain

$$\left.\begin{aligned} S(0) &= 142,028,910 \\ A(0) &= 33,024,476 \\ C(0) &= 10,000,000 \\ R(0) &= 1,000,000 \\ M(0) &= 200,000,000 \end{aligned}\right\} \tag{55}$$

$$\left.\begin{aligned} &S_0 = S(0) + \Lambda t \\ &S_{n+1} = -\frac{pc(1-\varepsilon)}{N}\int_0^t F_n dt + \omega\int_0^t R_n dt - \mu\int_0^t S_n dt,\ (for\, n \geq 0) \end{aligned}\right\} \tag{56}$$

$$\left.\begin{aligned} &A_0 = S(0) \\ &A_{n+1} = \frac{pc(1-\varepsilon)}{N}\int_0^t F_n dt - \left[\sigma_1(1-\rho) + \theta(1-\alpha) + \tau_1\alpha + \mu\right]\int_0^t A_n dt,\ (for\, n \geq 0) \end{aligned}\right\} \tag{57}$$

$$\left.\begin{aligned} &C_0 = C(0) \\ &C_{n+1} = \theta(1-\alpha)\int_0^t A_n dt - \left[\sigma_2(1-\rho) + \tau_2 + \mu\right]\int_0^t C_n dt,\ (for\, n \geq 0) \end{aligned}\right\} \tag{58}$$

$$\left.\begin{aligned} &R_0 = R(0) \\ &R_{n+1} = \tau_1\alpha\int_0^t A_n dt + \tau_2\int_0^t C_n dt - (\omega + \mu)\int_0^t R_n dt,\ (for\, n \geq 0) \end{aligned}\right\} \tag{59}$$

$$\left.\begin{aligned} &M_0 = M(0) \\ &M_{n+1} = \sigma_1(1-\rho)\int_0^t A_n dt + \sigma_2(1-\rho)\int_0^t C_n dt - \eta\int_0^t M_n dt,\ (for\, n \geq 0) \end{aligned}\right\} \tag{60}$$

The first nine (9) approximations of each compartment are computed and presented below:

$$S(t) = \begin{Bmatrix} 142028910 - 6.737541053\times10^6 t + 1.116840209\times10^7 t^2 - 2.822311883\times10^7 t^3 \\ +8.842934405\times10^7 t^4 - 3.387541995\times10^8 t^5 + 1.545032507\times10^9 t^6 \\ -8.186119763\times10^9 t^7 + 4.937968772\times10^{10} t^8 + 3.680704607\times10^8 t^9 + \ldots \end{Bmatrix} \tag{61}$$

$$A(t) = \begin{Bmatrix} 33024476 - 2.726263432\times10^7 t + 5.244854382\times10^7 t^2 - 1.515217243\times10^8 t^3 \\ +5.841640693\times10^8 t^4 - 2.818464469\times10^9 t^5 + 1.633527191\times10^{10} t^6 \\ -1.105505447\times10^{11} t^7 + 8.556320202\times10^{11} t^8 + 3.330804054\times10^9 t^9 + \ldots \end{Bmatrix} \tag{62}$$

$$C(t) = \begin{Bmatrix} 10000000 - 1.145104800\times10^6 t - 9.130141290\times10^6 t^2 + 5.269670480\times10^7 t^3 \\ -2.845771643\times10^8 t^4 + 1.686900581\times10^9 t^5 - 1.122624507\times10^{10} t^6 \\ +8.377176018\times10^{10} t^7 - 6.962657846\times10^{11} t^8 - 3.727377257\times10^9 t^9 + \ldots \end{Bmatrix} \tag{63}$$

$$R(t) = \begin{Bmatrix} 1000000 + 3.208935700\times10^7 t - 5.409959849\times10^7 t^2 + 1.265178901\times10^8 t^3 \\ -3.870618678\times10^8 t^4 + 1.469871266\times10^9 t^5 - 6.670679232\times10^9 t^6 \\ +3.518075201\times10^{10} t^7 - 2.110515441\times10^{11} t^8 - 9.716147980\times10^7 t^9 + \ldots \end{Bmatrix} \tag{64}$$

$$M(t)=\begin{cases}200000000+2.451223800\times10^{5}t-2.955774637\times10^{5}t^{2}+5.129125920\times10^{5}t^{3}\\-9.227714590\times10^{5}t^{4}+3.757048957\times10^{5}t^{5}+1.665987543\times10^{7}t^{6}\\-2.141142998\times10^{8}t^{7}+2.279890317\times10^{9}t^{8}+1.22375991410^{8}t^{9}+...\end{cases} \quad (65)$$

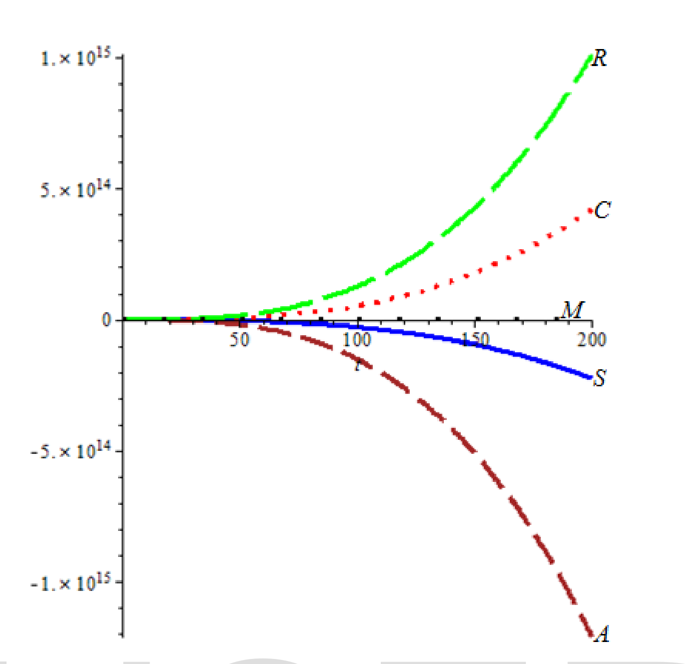

Figure 1: Plots of 3rd terms approximations for $S(t), A(t), C(t), R(t)$ *and* $M(t)$ against time (t)

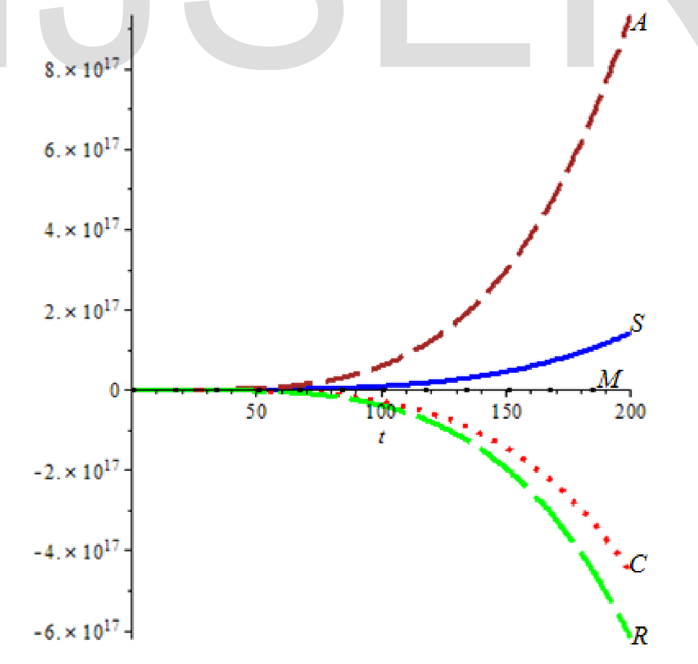

Figure 2: Plots of 4th terms approximations for $S(t), A(t), C(t), R(t)$ *and* $M(t)$ against time (t)

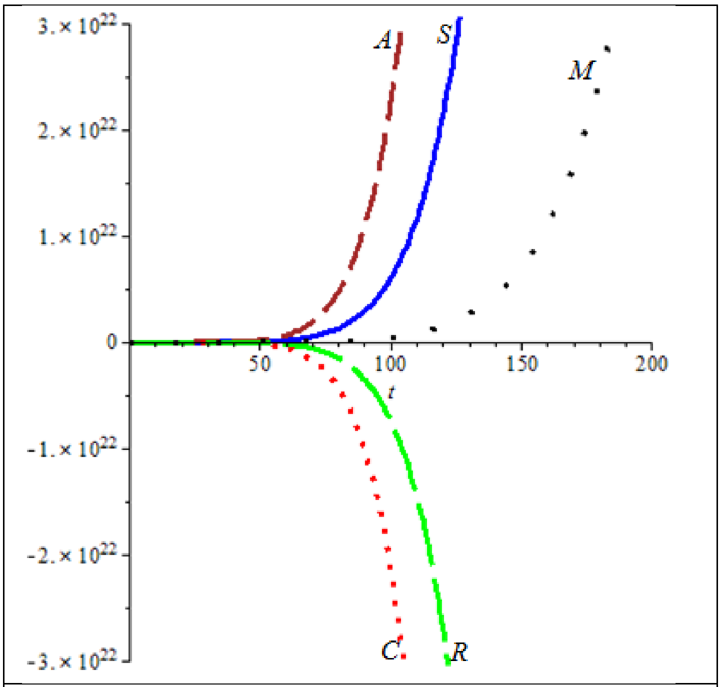

Figure 3: Plots of 7th terms approximations for $S(t), A(t), C(t), R(t)$ *and* $M(t)$ against time (t)

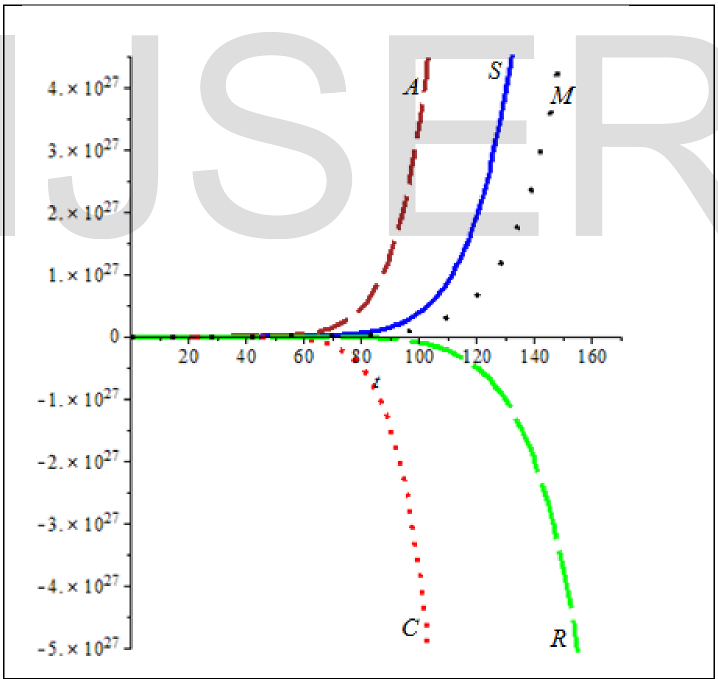

Figure 4: Plots of 9th terms approximations for $S(t), A(t), C(t), R(t)$ *and* $M(t)$ against time (t)

From Figure 1 – 4 it can be seen that the various order of approximations yielded convergent series that are reasonable and easy to express. The plots show that while the number of chronically infected individuals (C) decreases, the population of acutely infected individuals (A) increases in the period of the epidemic. Meanwhile, the number of microbes (M) in the toilets continue to increase, indicating the poor conditions of toilets in Nigeria. However, from figure 4, as toilet infection dies out $(i.e\ C \to 0)$, population in other compartments continue to increase and S

approaches some positive value $\left(i.e\ S = 3.680704607 \times 10^{8}\right)$ which is the eventual population who were never infected.

5. CONCLUSION

In this paper, the ADM has been successfully applied to approximately solve a system of nonlinear equations in the dynamics and control of SACRM toilet infection epidemic model. The result shows the potential efficiency of ADM in solving nonlinear problems. It can thus be concluded that when combined with high performance computer and symbolic computation software such as Maple, Mathlab, Mathematica and so on, the method stands a chance of becoming a new powerful analytic tool to obtaining a satisfactory non-linearized and unperturbed approximations for nonlinear problems with unrestrictive assumptions in the field of epidemiology, science and engineering at large.

6. RECOMMENDATIONS

The 7^{th} and 9^{th} terms approximations from figure 3 and 4 respectively indicated the increasing nature of the population of microbes in Nigerian toilets. This is due to the poor state of such toilets. It is therefore recommended that the government should ensure through the relevant bodies that every building that will be accommodating people have provision(s) for private and public toilets. Government should also ensure the provision of stable portable water because a toilet without stable water for maintenance is as good as not having one. Lastly, the government through the relevant bodies should encourage good sanitation and personal hygiene as this will reduce the spread of infections from toilets.

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